Patterns and Inductive Reasoning

Objective To use inductive reasoning to make conjectures

Getting Ready!
Fold a piece of paper in half. When you unfold it, the paper is divided into two rectangles. Refold the paper, and then fold it in half again. This time when you unfold it, there are four rectangles. How many rectangles would you get if you folded a piece of paper in half eight times? Explain.

In the Solve It, you may have used inductive reasoning. **Inductive reasoning** is reasoning based on patterns you observe.

**Essential Understanding** You can observe patterns in some number sequences and some sequences of geometric figures to discover relationships.

Lesson Vocabulary
- inductive reasoning
- conjecture
- counterexample

Plan How do you look for a pattern in a sequence? Look for a relationship between terms. Test that the relationship is consistent throughout the sequence.

Problem 1 Finding and Using a Pattern

Look for a pattern. What are the next two terms in each sequence?

A 3, 9, 27, 81, ...

\[\begin{align*}
3 & \quad 9 \\
\times 3 & \\
27 & \\
\times 3 & \\
81 & \\
\end{align*}\]

Each term is three times the previous term. The next two terms are

\[81 \times 3 = 243 \quad \text{and} \quad 243 \times 3 = 729.\]

B

Each circle contains a polygon that has one more side than the preceding polygon. The next two circles contain a six-sided and a seven-sided polygon.

Got It? 1. What are the next two terms in each sequence?

a. 45, 40, 35, 30, ...

b.
You may want to find the tenth or the one-hundredth term in a sequence. In this case, rather than find every previous term, you can look for a pattern and make a conjecture. A conjecture is a conclusion you reach using inductive reasoning.

**Problem 2 Using Inductive Reasoning**

Look at the circles. What conjecture can you make about the number of regions 20 diameters form?

1 diameter forms 2 regions.
2 diameters form 4 regions.
3 diameters form 6 regions.

Each circle has twice as many regions as diameters. Twenty diameters form $20 \cdot 2$, or 40 regions.

**Got It?** 2. What conjecture can you make about the twenty-first term in $R, W, B, R, W, B, \ldots$?

It is important to gather enough data before you make a conjecture. For example, you do not have enough information about the sequence $1, 3, \ldots$ to make a reasonable conjecture. The next term could be $3 \cdot 3 = 9$ or $3 + 2 = 5$.

**Problem 3 Collecting Information to Make a Conjecture**

What conjecture can you make about the sum of the first 30 even numbers?

Find the first few sums and look for a pattern.

<table>
<thead>
<tr>
<th>Number of Terms</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>$2 + 4$ = 6</td>
</tr>
<tr>
<td>3</td>
<td>$2 + 4 + 6$ = 12</td>
</tr>
<tr>
<td>4</td>
<td>$2 + 4 + 6 + 8$ = 20</td>
</tr>
</tbody>
</table>

Each sum is the product of the number of terms and the number of terms plus one.

You can conclude that the sum of the first 30 even numbers is $30 \cdot 31$, or 930.

**Got It?** 3. What conjecture can you make about the sum of the first 30 odd numbers?
**Problem 4  Making a Prediction**

**Sales** Sales of backpacks at a nationwide company decreased over a period of six consecutive months. What conjecture can you make about the number of backpacks the company will sell in May?

The points seem to fall on a line. The graph shows the number of sales decreasing by about 500 backpacks each month. By inductive reasoning, you can estimate that the company will sell approximately 8000 backpacks in May.

![Graph showing Backpacks Sold over months]

**Got It?** 4. a. What conjecture can you make about backpack sales in June?

b. **Reasoning** Is it reasonable to use this graph to make a conjecture about sales in August? Explain.

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Not all conjectures turn out to be true. You should test your conjecture multiple times. You can prove that a conjecture is false by finding one counterexample. A **counterexample** is an example that shows that a conjecture is incorrect.

**Problem 5  Finding a Counterexample**

What is a counterexample for each conjecture?

A. If the name of a month starts with the letter J, it is a summer month.

   Counterexample: January starts with J and it is a winter month.

B. You can connect any three points to form a triangle.

   Counterexample: If the three points lie on a line, you cannot form a triangle.

   ![Three points forming a triangle and counterexample]

   These three points support the conjecture…

   …but these three points are a counterexample to the conjecture.

C. When you multiply a number by 2, the product is greater than the original number.

   The conjecture is true for positive numbers, but it is false for negative numbers and zero.

   Counterexample: \(-4 \cdot 2 = -8\) and \(-8 \neq -4\).

**Got It?** 5. What is a counterexample for each conjecture?

a. If a flower is red, it is a rose.

b. One and only one plane exists through any three points.

c. When you multiply a number by 3, the product is divisible by 6.
Lesson Check

Do you know HOW?

What are the next two terms in each sequence?

1. 7, 13, 19, 25, . . .

2. [Diagram of colored shapes]

3. What is a counterexample for the following conjecture?
   All four-sided figures are squares.

Do you UNDERSTAND?

4. **Vocabulary** How does the word *counter* help you understand the term *counterexample*?

5. **Compare and Contrast** Clay thinks the next term in the sequence 2, 4, . . . is 6. Given the same pattern, Ott thinks the next term is 8, and Stacie thinks the next term is 7. What conjecture is each person making? Is there enough information to decide who is correct?

Practice and Problem-Solving Exercises

**A Practice**

Find a pattern for each sequence. Use the pattern to show the next two terms.

6. 5, 10, 20, 40, . . .

7. 1, 4, 9, 16, 25, . . .

8. 1, −1, 2, −2, 3, . . .

9. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

10. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

11. 15, 12, 9, 6, . . .


14. 1, 2, 6, 24, 120, . . .


16. Dollar coin, half dollar, quarter, . . .

17. AL, AK, AZ, AR, CA, . . .

18. Aquarius, Pisces, Aries, Taurus, . . .

19. [Diagram of colored shapes]

20. [Diagram of shapes]

Use the sequence and inductive reasoning to make a conjecture.

21. What is the color of the fifteenth figure?

22. What is the shape of the twelfth figure?

23. What is the color of the thirtieth figure?

24. What is the shape of the fortieth figure?

**Make a conjecture for each scenario. Show your work.**

25. the sum of the first 100 positive odd numbers

26. the sum of the first 100 positive even numbers

27. the sum of two odd numbers

28. the sum of an even and odd number

29. the product of two even numbers

30. the product of two odd numbers

PowerGeometry.com  Lesson 2.1 Patterns and Inductive Reasoning
Weather  Use inductive reasoning to make a prediction about the weather.

31. Lightning travels much faster than thunder, so you see lightning before you hear thunder. If you count 5 s between the lightning and thunder, how far away is the storm?

32. The speed at which a cricket chirps is affected by the temperature. If you hear 20 cricket chirps in 14 s, what is the temperature?

<table>
<thead>
<tr>
<th>Number of Chirps per 14 Seconds</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
</tr>
<tr>
<td>15</td>
<td>65</td>
</tr>
</tbody>
</table>

Find one counterexample to show that each conjecture is false.

33. \( \angle 1 \) and \( \angle 2 \) are supplementary, so one of the angles is acute.
34. \( \triangle ABC \) is a right triangle, so \( \angle A \) measures 90.
35. The sum of two numbers is greater than either number.
36. The product of two positive numbers is greater than either number.
37. The difference of two integers is less than either integer.

Apply

Find a pattern for each sequence. Use inductive reasoning to show the next two terms.

38. 1, 3, 7, 13, 21, \ldots
39. 1, 2, 5, 6, 9, \ldots
40. 0.1, 0.01, 0.001, \ldots
41. 2, 6, 7, 21, 22, 66, 67, \ldots
42. 1, 3, 7, 15, 31, \ldots
43. \( \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots \)

Predict the next term in each sequence. Use your calculator to verify your answer.

44. \( 12345679 \times 9 = 11111111 \)
\( 12345679 \times 18 = 22222222 \)
\( 12345679 \times 27 = 33333333 \)
\( 12345679 \times 36 = 44444444 \)
\( 12345679 \times 45 = \) [to be calculated]

45. \( 1 \times 1 = 1 \)
\( 11 \times 11 = 121 \)
\( 111 \times 111 = 12321 \)
\( 1111 \times 1111 = 1234321 \)
\( 11111 \times 11111 = \) [to be calculated]

46. Patterns  Draw the next figure in the sequence. Make sure you think about color and shape.
47. Draw the next figure in each sequence.

48. 

49. **Reasoning** Find the perimeter when 100 triangles are put together in the pattern shown. Assume that all triangle sides are 1 cm long.

50. **Think About a Plan** Below are 15 points. Most of the points fit a pattern. Which does not? Explain.

   \[A(6, -2)\]  \[B(6, 5)\]  \[C(8, 0)\]  \[D(8, 7)\]  \[E(10, 2)\]  \[F(10, 6)\]  \[G(11, 4)\]  \[H(12, 3)\]  \[I(4, 0)\]  \[J(7, 6)\]  \[K(5, 6)\]  \[L(4, 7)\]  \[M(2, 2)\]  \[N(1, 4)\]  \[O(2, 6)\]

   - How can you draw a diagram to help you find a pattern?
   - What pattern do the majority of the points fit?

51. **Language** Look for a pattern in the Chinese number system.

   a. What is the Chinese name for the numbers 43, 67, and 84?

   b. **Reasoning** Do you think that the Chinese number system is base 10? Explain.

52. **Open-Ended** Write two different number sequences that begin with the same two numbers.

53. **Error Analysis** For each of the past four years, Paulo has grown 2 in. every year. He is now 16 years old and is 5 ft 10 in. tall. He figures that when he is 22 years old he will be 6 ft 10 in. tall. What would you tell Paulo about his conjecture?

54. **Bird Migration** During bird migration, volunteers get up early on Bird Day to record the number of bird species they observe in their community during a 24-h period. Results are posted online to help scientists and students track the migration.

   a. Make a graph of the data.

   b. Use the graph and inductive reasoning to make a conjecture about the number of bird species the volunteers in this community will observe in 2015.

55. **Writing** Describe a real-life situation in which you recently used inductive reasoning.
56. **History**  When he was in the third grade, German mathematician Karl Gauss (1777–1855) took ten seconds to sum the integers from 1 to 100. Now it’s your turn. Find a fast way to sum the integers from 1 to 100. Find a fast way to sum the integers from 1 to \( n \). (Hint: Use patterns.)

57. **Chess**  The small squares on a chessboard can be combined to form larger squares. For example, there are sixty-four \( 1 \times 1 \) squares and one \( 8 \times 8 \) square. Use inductive reasoning to determine how many \( 2 \times 2 \) squares, \( 3 \times 3 \) squares, and so on, are on a chessboard. What is the total number of squares on a chessboard?

58. a. **Algebra**  Write the first six terms of the sequence that starts with 1, and for which the difference between consecutive terms is first 2, and then 3, 4, 5, and 6.

b. Evaluate \( \frac{n^2 + n}{2} \) for \( n = 1, 2, 3, 4, 5, \) and 6. Compare the sequence you get with your answer for part (a).

c. Examine the diagram at the right and explain how it illustrates a value of \( \frac{n^2 + n}{2} \).

d. Draw a similar diagram to represent \( \frac{n^2 + n}{2} \) for \( n = 5 \).

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**Standardized Test Prep**

59. What is the next term in the sequence 1, 1, 2, 3, 5, 8, 13, . . . ?

- A. 17
- B. 20
- C. 21
- D. 24

60. A horse trainer wants to build three adjacent rectangular corrals as shown at the right. The area of each corral is 7200 ft\(^2\). If the length of each corral is 120 ft, how much fencing does the horse trainer need to buy in order to build the corrals?

- F. 300 ft
- G. 360 ft
- H. 560 ft
- I. 840 ft

61. The coordinates \( x, y, a, \) and \( b \) are all positive integers. Could the points \( (x, y) \) and \( (a, b) \) have a midpoint in Quadrant III? Explain.

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**Mixed Review**

62. What is the area of a circle with radius 4 in.? Leave your answer in terms of \( \pi \).

63. What is the perimeter of a rectangle with side lengths 3 m and 7 m?

64. Solve for \( x \) if \( B \) is the midpoint of \( \overline{AC} \).

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**Get Ready!**  To prepare for Lesson 2-2, do Exercises 65 and 66.

Tell whether each conjecture is **true** or **false**. Explain.

65. The sum of two even numbers is even.

66. The sum of three odd numbers is odd.
The study of *if-then* statements and their truth values is a foundation of reasoning.

**Essential Understanding** You can describe some mathematical relationships using a variety of *if-then* statements.

The Venn diagram above illustrates how the set of things that satisfy the hypothesis lies inside the set of things that satisfy the conclusion.
**Problem 1** Identifying the Hypothesis and the Conclusion

What are the hypothesis and the conclusion of the conditional?

If an animal is a robin, then the animal is a bird.

Hypothesis \((p)\): An animal is a robin.
Conclusion \((q)\): The animal is a bird.

**Got It?** 1. What are the hypothesis and the conclusion of the conditional?

If an angle measures 130, then the angle is obtuse.

**Problem 2** Writing a Conditional

How can you write the following statement as a conditional?

Vertical angles share a vertex.

**Step 1** Identify the hypothesis and the conclusion.

Vertical angles share a vertex.

**Step 2** Write the conditional.

If two angles are vertical, then they share a vertex.

**Got It?** 2. How can you write “Dolphins are mammals” as a conditional?

**Problem 3** Finding the Truth Value of a Conditional

Is the conditional true or false? If it is false, find a counterexample.

A If a woman is Hungarian, then she is European.

The conditional is true. Hungary is a European nation, so Hungarians are European.

B If a number is divisible by 3, then it is odd.

The conditional is false. The number 12 is divisible by 3, but it is not odd.

**Got It?** 3. Is the conditional true or false? If it is false, find a counterexample.

a. If a month has 28 days, then it is February.
   b. If two angles form a linear pair, then they are supplementary.
The **negation** of a statement \( p \) is the opposite of the statement. The symbol is \( \sim p \) and is read “not \( p \).” The negation of the statement “The sky is blue” is “The sky is *not* blue.” You can use negations to write statements related to a conditional. Every conditional has three related conditional statements.

### Key Concept: Related Conditional Statements

<table>
<thead>
<tr>
<th>Statement</th>
<th>How to Write It</th>
<th>Example</th>
<th>Symbols</th>
<th>How to Read It</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional</strong></td>
<td>Use the given hypothesis and conclusion.</td>
<td>If ( m \angle A = 15 ), then ( \angle A ) is acute.</td>
<td>( p \rightarrow q )</td>
<td>If ( p ), then ( q ).</td>
</tr>
<tr>
<td><strong>Converse</strong></td>
<td>Exchange the hypothesis and the conclusion.</td>
<td>If ( \angle A ) is acute, then ( m \angle A = 15 ).</td>
<td>( q \rightarrow p )</td>
<td>If ( q ), then ( p ).</td>
</tr>
<tr>
<td><strong>Inverse</strong></td>
<td>Negate both the hypothesis and the conclusion of the conditional.</td>
<td>If ( m \angle A \neq 15 ), then ( \angle A ) is not acute.</td>
<td>( \sim p \rightarrow \sim q )</td>
<td>If not ( p ), then not ( q ).</td>
</tr>
<tr>
<td><strong>Contrapositive</strong></td>
<td>Negate both the hypothesis and the conclusion of the converse.</td>
<td>If ( \angle A ) is not acute, then ( m \angle A \neq 15 ).</td>
<td>( \sim q \rightarrow \sim p )</td>
<td>If not ( q ), then not ( p ).</td>
</tr>
</tbody>
</table>

Below are the truth values of the related statements above. **Equivalent statements** have the same truth value.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Example</th>
<th>Truth Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional</strong></td>
<td>If ( m \angle A = 15 ), then ( \angle A ) is acute.</td>
<td>True</td>
</tr>
<tr>
<td><strong>Converse</strong></td>
<td>If ( \angle A ) is acute, then ( m \angle A = 15 ).</td>
<td>False</td>
</tr>
<tr>
<td><strong>Inverse</strong></td>
<td>If ( m \angle A \neq 15 ), then ( \angle A ) is not acute.</td>
<td>False</td>
</tr>
<tr>
<td><strong>Contrapositive</strong></td>
<td>If ( \angle A ) is not acute, then ( m \angle A \neq 15 ).</td>
<td>True</td>
</tr>
</tbody>
</table>

A conditional and its contrapositive are equivalent statements. They are either both true or both false. The converse and inverse of a statement are also equivalent statements.
Problem 4  Writing and Finding Truth Values of Statements

What are the converse, inverse, and contrapositive of the following conditional? What are the truth values of each? If a statement is false, give a counterexample.

If the figure is a square, then the figure is a quadrilateral.

Think
Identify the hypothesis and the conclusion.

To write the converse, switch the hypothesis and the conclusion. Write \( q \rightarrow p \).

To write the inverse, negate both the hypothesis and the conclusion of the conditional. Write \( \neg p \rightarrow \neg q \).

To write the contrapositive, negate both the hypothesis and the conclusion of the converse. Write \( \neg q \rightarrow \neg p \).

Write

\( p \): The figure is a square.
\( q \): The figure is a quadrilateral.

Converse: If the figure is a quadrilateral, then the figure is a square.
The converse is false. Counterexample: A rectangle that is not a square.
Inverse: If the figure is not a square, then the figure is not a quadrilateral.
The inverse is false. Counterexamples:

Contrapositive: If the figure is not a quadrilateral, then the figure is not a square. The contrapositive is true.

Got It? 4. What are the converse, inverse, and contrapositive of the conditional statement below? What are the truth values of each? If a statement is false, give a counterexample.

If a vegetable is a carrot, then it contains beta carotene.

Lesson Check

Do you know HOW?

1. What are the hypothesis and the conclusion of the following statement? Write it as a conditional.
   Residents of Key West live in Florida.

2. What are the converse, inverse, and contrapositive of the statement? Which statements are true?
   If a figure is a rectangle with sides 2 cm and 3 cm, then it has a perimeter of 10 cm.

Do you UNDERSTAND?

3. Error Analysis Your classmate rewrote the statement “You jog every Sunday” as the following conditional. What is your classmate’s error? Correct it.
   If you jog, then it is Sunday.

4. Reasoning Suppose a conditional statement and its converse are both true. What are the truth values of the contrapositive and inverse? How do you know?
Practice and Problem-Solving Exercises

Practice

1. Identify the hypothesis and conclusion of each conditional.
2. If you are an American citizen, then you have the right to vote.
3. If a figure is a rectangle, then it has four sides.
4. If you want to be healthy, then you should eat vegetables.

Write each sentence as a conditional.
5. Hank Aaron broke Babe Ruth’s home-run record.
6. Algebra: \[ 3x - 7 = 14 \] implies that \[ 3x = 21 \].
7. Thanksgiving in the United States falls on the fourth Thursday of November.
8. A counterexample shows that a conjecture is false.
9. Coordinate Geometry: A point in the first quadrant has two positive coordinates.

Write a conditional statement that each Venn diagram illustrates.

10. [Diagram: Colors, Blue]
11. [Diagram: Integers, Whole numbers]
12. [Diagram: Grains, Wheat]

Determine if the conditional is true or false. If it is false, find a counterexample.
13. If a polygon has eight sides, then it is an octagon.
14. If you live in a country that borders the United States, then you live in Canada.
15. If you play a sport with a ball and a bat, then you play baseball.
16. If an angle measures 80, then it is acute.

If the given statement is not in if-then form, rewrite it. Write the converse, inverse, and contrapositive of the given conditional statement. Determine the truth value of all four statements. If a statement is false, give a counterexample.
17. If you are a quarterback, then you play football.
18. Pianists are musicians.
19. Algebra: If \[ 4x + 8 = 28 \], then \( x = 5 \).
20. Odd natural numbers less than 8 are prime.
21. Two lines that lie in the same plane are coplanar.
Write each statement as a conditional.

25. “We’re half the people; we should be half the Congress.” —Jeanette Rankin, former U.S. congresswoman, calling for more women in office

26. “Anyone who has never made a mistake has never tried anything new.”
   —Albert Einstein

27. **Probability** An event with probability 1 is certain to occur.

28. **Think About a Plan** Your classmate claims that the conditional and contrapositive of the following statement are both true. Is he correct? Explain.
   - If \( x = 2 \), then \( x^2 = 4 \).
   - Can you find a counterexample of the conditional?
   - Do you need to find a counterexample of the contrapositive to know its truth value?

29. **Open-Ended** Write a true conditional that has a true converse, and write a true conditional that has a false converse.

30. **Multiple Representations** Write three separate conditional statements that the Venn diagram illustrates.

31. **Error Analysis** A given conditional is true. Natalie claims its contrapositive is also true. Sean claims its contrapositive is false. Who is correct and how do you know?

   Draw a Venn diagram to illustrate each statement.

32. If an angle measures 100°, then it is obtuse.

33. If you are the captain of your team, then you are a junior or senior.

34. Peace Corps volunteers want to help other people.

**Algebra** Write the converse of each statement. If the converse is true, write true. If it is not true, provide a counterexample.

35. If \( x = -6 \), then \( |x| = 6 \).

36. If \( y \) is negative, then \( -y \) is positive.

37. If \( x < 0 \), then \( x^3 < 0 \).

38. If \( x < 0 \), then \( x^2 > 0 \).

39. **Advertising** Advertisements often suggest conditional statements. What conditional does the ad at the right imply?

**Look Cool!**

**Wear Snazzy Sneakers**

Write each postulate as a conditional statement.

40. Two intersecting lines meet in exactly one point.

41. Two congruent figures have equal areas.

42. Through any two points there is exactly one line.
Write a statement beginning with *all, some, or no* to match each Venn diagram.

43. [Integers divisible by 2, Integers divisible by 8, Triangles, Squares, Students, Musicians]

46. Let \( a \) represent an integer. Consider the five statements \( r, s, t, u, \) and \( v \).

- \( r: a \) is even.
- \( s: a \) is odd.
- \( t: 2a \) is even.
- \( u: 2a \) is odd.
- \( v: 2a + 1 \) is odd.

How many statements of the form \( p \rightarrow q \) can you make from these statements? Decide which are true, and provide a counterexample if they are false.

### Standardized Test Prep

47. Which conditional and its converse are both true?

- **A** If \( x = 1 \), then \( 2x = 2 \).
- **B** If \( x = 2 \), then \( x^2 = 4 \).
- **C** If \( x = 3 \), then \( x^2 = 6 \).
- **D** If \( x^2 = 4 \), then \( x = 2 \).

48. What is the midpoint of the segment with endpoints \((-3, 7)\) and \((9, 5)\)?

- **A** \((6, 12)\)
- **B** \((2, 4)\)
- **C** \((3, 6)\)
- **D** \((6, 6)\)

49. Which is the best description of the figure at the right?

- **A** convex pentagon
- **B** concave octagon
- **C** convex polygon
- **D** concave pentagon

50. Describe how to form the Fibonacci sequence \(1, 1, 2, 3, 5, 8, 13, \ldots\)

### Mixed Review

Find a counterexample to show that each statement is false.

51. You can connect any four points to form a rectangle.

52. The square of a number is always greater than the number.

Find the perimeter of each rectangle with the given base and height.

53. 6 in., 12 in.  
54. 3.5 cm, 7 cm  
55. \( 12, \frac{3}{4} \) yd, 18 in.  
56. 11 m, 60 cm

### Get Ready!

To prepare for Lesson 2-3, do Exercises 57 and 58.

Write the converse of each statement. Then determine the truth value of the original statement and of the converse.

57. If today is September 30, then tomorrow is October 1.

58. If \( \overline{AB} \) is the perpendicular bisector of \( \overline{CD} \), then \( \overline{AB} \) and \( \overline{CD} \) are perpendicular.
A **compound statement** combines two or more statements.

### Key Concepts: Compound Statements

<table>
<thead>
<tr>
<th>Compound Statement</th>
<th>How to Form It</th>
<th>Example</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>conjunction</strong></td>
<td>Connect two or more statements with <em>and.</em></td>
<td>You will eat a sandwich and you will drink juice.</td>
<td>$s \land j$ You say “$s$ and $j$.”</td>
</tr>
<tr>
<td><strong>disjunction</strong></td>
<td>Connect two or more statements with <em>or.</em></td>
<td>You will eat a sandwich or you will drink juice.</td>
<td>$s \lor j$ You say “$s$ or $j$.”</td>
</tr>
</tbody>
</table>

A conjunction $s \land j$ is true only when both $s$ and $j$ are true.

A disjunction $s \lor j$ is false only when both $s$ and $j$ are false.

### Activity 1

For Exercises 1–4, use the statements below to construct the following compound statements.

$s$: We will go to the beach.
$j$: We will go out to dinner.
$t$: We will go to the movies.

1. $s \land j$
2. $s \lor j$
3. $s \lor (j \land t)$
4. $(s \lor j) \land t$

5. Write three of your own statements and label them $s$, $j$, and $t$. Repeat Exercises 1–4 using your own statements.

For Exercises 6–9, use the statements below to determine the truth value of the compound statement.

$x$: Emperor penguins are black and white.
$y$: Polar bears are a threatened species.
$z$: Penguins wear tuxedos.

6. $x \land y$
7. $x \lor y$
8. $x \land z$
9. $x \lor z$
A truth table lists all the possible combinations of truth values for two or more statements.

<table>
<thead>
<tr>
<th>Example</th>
<th>( p )</th>
<th>( q )</th>
<th>( p \rightarrow q )</th>
<th>( p \land \neg q )</th>
<th>( p \lor q )</th>
</tr>
</thead>
</table>
| \( p \): Ohio is a state.  
\( q \): There are 50 states. | T   | T   | T               | T               | T               |
| \( p \): Georgia is a state.  
\( q \): Miami is a state. | T   | F   | F               | F               | T               |
| \( p \): \( 2 + 2 = 5 \)  
\( q \): \( 2 \cdot 2 = 4 \) | F   | T   | T               | F               | T               |
| \( p \): \( 2 + 1 = 4 \)  
\( q \): Dolphins are big fish. | F   | F   | T               | F               | F               |

**Activity 2**

To find the possible truth values of a complex statement such as \( (s \land f) \lor \neg t \), you can make a truth table like the one below. You start with columns for the single statements and add columns to the right. Each column builds toward the final statement. The table below starts with columns for \( s, f, \) and \( t \) and builds to \( (s \land f) \lor \neg t \). Copy the table and work with a partner to fill in the blanks.

<table>
<thead>
<tr>
<th>( s )</th>
<th>( j )</th>
<th>( t )</th>
<th>( \neg t )</th>
<th>( s \land f )</th>
<th>( (s \land f) \lor \neg t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>\cellcolor{green}20. T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>\cellcolor{red}13. F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>\cellcolor{green}21. T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>\cellcolor{red}17. F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>\cellcolor{red}14. F</td>
<td>F</td>
<td>\cellcolor{green}22. T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>\cellcolor{red}15. F</td>
<td>F</td>
<td>\cellcolor{green}23. F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>\cellcolor{red}18. F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>\cellcolor{green}10. T</td>
<td>\cellcolor{green}11. T</td>
<td>\cellcolor{green}12. T</td>
<td>\cellcolor{green}16. T</td>
<td>\cellcolor{green}19. T</td>
<td>\cellcolor{green}24. T</td>
</tr>
</tbody>
</table>

25. Make a truth table for the statement \( (\neg p \lor q) \land \neg r \).
26. **a.** Make a truth table for \( \neg(p \land q) \). Make another truth table for \( \neg p \lor \neg q \).
   **b.** Make a truth table for \( \neg(p \lor q) \). Make another for \( \neg p \land \neg q \).
   **c.** DeMorgan’s Law states that \( \neg(p \lor q) = \neg p \land \neg q \) and that \( \neg(p \land q) = \neg p \lor \neg q \). How do the truth tables you made in parts (a) and (b) show that DeMorgan’s Law is true?
Biconditionals and Definitions

Objective  To write biconditionals and recognize good definitions

In the Solve It, you used conditional statements. A **biconditional** is a single true statement that combines a true conditional and its true converse. You can write a biconditional by joining the two parts of each conditional with the phrase *if and only if*.

**Essential Understanding**  A definition is good if it can be written as a biconditional.

**Problem 1  Writing a Biconditional**

What is the converse of the following true conditional? If the reverse is also true, rewrite the statements as a biconditional.

If the sum of the measures of two angles is 180, then the two angles are supplementary.

**Converse**: If two angles are supplementary, then the sum of the measures of the two angles is 180.

The converse is true. You can form a true biconditional by joining the true conditional and the true converse with the phrase *if and only if*.

**Biconditional**: Two angles are supplementary if and only if the sum of the measures of the two angles is 180.

**Got It?**  1. What is the converse of the following true conditional? If the converse is also true, rewrite the statements as a biconditional.

   If two angles have equal measure, then the angles are congruent.
Key Concept  Biconditional Statements

A biconditional combines \( p \rightarrow q \) and \( q \rightarrow p \) as \( p \leftrightarrow q \).

<table>
<thead>
<tr>
<th>Example</th>
<th>Symbols</th>
<th>How to Read It</th>
</tr>
</thead>
<tbody>
<tr>
<td>A point is a midpoint if and only if it divides a segment into two congruent segments.</td>
<td>( p \leftrightarrow q )</td>
<td>&quot;( p ) if and only if ( q )&quot;</td>
</tr>
</tbody>
</table>

You can write a biconditional as two conditionals that are converses.

**Problem 2  Identifying the Conditionals in a Biconditional**

What are the two conditional statements that form this biconditional?
- A ray is an angle bisector if and only if it divides an angle into two congruent angles.

Let \( p \) and \( q \) represent the following:
- \( p \): A ray is an angle bisector.
- \( q \): A ray divides an angle into two congruent angles.

- \( p \rightarrow q \): If a ray is an angle bisector, then it divides an angle into two congruent angles.
- \( q \rightarrow p \): If a ray divides an angle into two congruent angles, then it is an angle bisector.

**Got It?  2.** What are the two conditionals that form this biconditional?
- Two numbers are reciprocals if and only if their product is 1.

As you learned in Lesson 1-2, undefined terms such as point, line, and plane are the building blocks of geometry. You understand the meanings of these terms intuitively. Then you use them to define other terms such as segment.

A good definition is a statement that can help you identify or classify an object. A good definition has several important components.

- ✔ A good definition uses clearly understood terms. These terms should be commonly understood or already defined.
- ✔ A good definition is precise. Good definitions avoid words such as large, sort of, and almost.
- ✔ A good definition is reversible. That means you can write a good definition as a true biconditional.
**Problem 3** Writing a Definition as a Biconditional

Is this definition of quadrilateral reversible? If yes, write it as a true biconditional.

Definition: A quadrilateral is a polygon with four sides.

**Think**

Write a conditional.

Write the converse.

The conditional and its converse are both true. The definition is reversible. Write the conditional and its converse as a true biconditional.

**Write**

**Conditional:** If a figure is a quadrilateral, then it is a polygon with four sides.

**Converse:** If a figure is a polygon with four sides, then it is a quadrilateral.

**Biconditional:** A figure is a quadrilateral if and only if it is a polygon with four sides.

**Got It?** 3. Is this definition of straight angle reversible? If yes, write it as a true biconditional.

A straight angle is an angle that measures 180.

One way to show that a statement is not a good definition is to find a counterexample.

**Problem 4** Identifying Good Definitions

**Multiple Choice** Which of the following is a good definition?

- **A** A fish is an animal that swims.
- **B** Rectangles have four corners.
- **C** Giraffes are animals with very long necks.
- **D** A penny is a coin worth one cent.

Choice A is not reversible. A whale is a counterexample. A whale is an animal that swims, but it is a mammal, not a fish. In Choice B, corners is not clearly defined. All quadrilaterals have four corners. In Choice C, very long is not precise. Also, Choice C is not reversible because ostriches also have long necks. Choice D is a good definition. It is reversible, and all of the terms in the definition are clearly defined and precise. The answer is D.

**Got It?** 4. a. Is the following statement a good definition? Explain.

A square is a figure with four right angles.

b. **Reasoning** How can you rewrite the statement “Obtuse angles have greater measures than acute angles” so that it is a good definition?
Lesson Check

Do you know HOW?

1. How can you write the following statement as two true conditionals?
   Collinear points are points that lie on the same line.

2. How can you combine the following statements as a biconditional?
   If this month is June, then next month is July.
   If next month is July, then this month is June.

3. Write the following definition as a biconditional.
   Vertical angles are two angles whose sides are opposite rays.

Do you UNDERSTAND?

4. Vocabulary Explain how the term biconditional is fitting for a statement composed of two conditionals.

5. Error Analysis Why is the following statement a poor definition?
   Elephants are gigantic animals.

6. Compare and Contrast Which of the following statements is a better definition of a linear pair? Explain.
   A linear pair is a pair of supplementary angles.
   A linear pair is a pair of adjacent angles with noncommon sides that are opposite rays.

Practice and Problem-Solving Exercises

Practice

Each conditional statement below is true. Write its converse. If the converse is also true, combine the statements as a biconditional.

7. If two segments have the same length, then they are congruent.

8. Algebra If \( x = 12 \), then \( 2x - 5 = 19 \).

9. If a number is divisible by 20, then it is even.

10. Algebra If \( x = 3 \), then \( |x| = 3 \).

11. In the United States, if it is July 4, then it is Independence Day.

12. If \( p \rightarrow q \) is true, then \( \sim q \rightarrow \sim p \) is true.

Write the two statements that form each biconditional.

13. A line bisects a segment if and only if the line intersects the segment only at its midpoint.

14. An integer is divisible by 100 if and only if its last two digits are zeros.

15. You live in Washington, D.C., if and only if you live in the capital of the United States.

16. A polygon is a triangle if and only if it has exactly three sides.

17. An angle is a right angle if and only if it measures 90.

18. Algebra \( x^2 = 144 \) if and only if \( x = 12 \) or \( x = -12 \).
Test each statement below to see if it is reversible. If so, write it as a true biconditional. If not, write not reversible.

19. A perpendicular bisector of a segment is a line, segment, or ray that is perpendicular to a segment at its midpoint.

20. Complementary angles are two angles with measures that have a sum of 90.

21. A Tarheel is a person who was born in North Carolina.

22. A rectangle is a four-sided figure with at least one right angle.

23. Two angles that form a linear pair are adjacent.

Is each statement below a good definition? If not, explain.

24. A cat is an animal with whiskers.

25. The red wolf is an endangered animal.

26. A segment is part of a line.

27. A compass is a geometric tool.

28. Opposite rays are two rays that share the same endpoint.

29. Perpendicular lines are two lines that intersect to form right angles.

30. **Think About a Plan** Is the following a good definition? Explain.
   A ligament is a band of tough tissue connecting bones or holding organs in place.
   • Can you write the statement as two true conditionals?
   • Are the two true conditionals converses of each other?

31. **Reasoning** Is the following a good definition? Explain.
   An obtuse angle is an angle with measure greater than 90.

32. **Open-Ended** Choose a definition from a dictionary or from a glossary. Explain what makes the statement a good definition.

33. **Error Analysis** Your friend defines a right angle as an angle that is greater than an acute angle. Use a biconditional to show that this is not a good definition.

34. Which conditional and its converse form a true biconditional?
   • If \( x > 0 \), then \( |x| > 0 \).
   • If \( x^3 = 5 \), then \( x = 125 \).
   • If \( x = 3 \), then \( x^2 = 9 \).
   • If \( x = 19 \), then \( 2x - 3 = 35 \).

Write each statement as a biconditional.

35. Points in Quadrant III have two negative coordinates.

36. When the sum of the digits of an integer is divisible by 9, the integer is divisible by 9 and vice versa.

37. The whole numbers are the nonnegative integers.

38. A hexagon is a six-sided polygon.
Language  For Exercises 39–42, use the chart below. Decide whether the description of each letter is a good definition. If not, provide a counterexample by giving another letter that could fit the definition.

39. The letter $D$ is formed by pointing straight up with the finger beside the thumb and folding the other fingers and the thumb so that they all touch.

40. The letter $K$ is formed by making a $V$ with the two fingers beside the thumb.

41. You have formed the letter $I$ if and only if the smallest finger is sticking up and the other fingers are folded into the palm of your hand with your thumb folded over them and your hand is held still.

42. You form the letter $B$ by holding all four fingers tightly together and pointing them straight up while your thumb is folded into the palm of your hand.

Reading Math  Let statements $p$, $q$, $r$, and $s$ be as follows:

$p$: $\angle A$ and $\angle B$ are a linear pair.
$q$: $\angle A$ and $\angle B$ are supplementary angles.
$r$: $\angle A$ and $\angle B$ are adjacent angles.
$s$: $\angle A$ and $\angle B$ are adjacent and supplementary angles.

Substitute for $p$, $q$, $r$, and $s$, and write each statement the way you would read it.

43. $p \rightarrow q$
44. $p \rightarrow r$
45. $p \rightarrow s$
46. $p \leftrightarrow s$

Challenge 47. Writing  Use the figures to write a good definition of a line in spherical geometry.
48. **Multiple Representations** You have illustrated true conditional statements with Venn diagrams. You can do the same thing with true biconditionals. Consider the following statement.

An integer is divisible by 10 if and only if its last digit is 0.

a. Write the two conditional statements that make up this biconditional.
b. Illustrate the first conditional from part (a) with a Venn diagram.
c. Illustrate the second conditional from part (a) with a Venn diagram.
d. Combine your two Venn diagrams from parts (b) and (c) to form a Venn diagram representing the biconditional statement.
e. What must be true of the Venn diagram for any true biconditional statement?
f. **Reasoning** How does your conclusion in part (e) help to explain why you can write a good definition as a biconditional?

---

**Standardized Test Prep**

49. Which statement is a good definition?

- A. Rectangles are usually longer than they are wide.
- B. Squares are convex.
- C. Circles have no corners.
- D. Triangles are three-sided polygons.

---

50. What is the exact area of a circle with a diameter of 6 cm?

- F. 28.27 cm
- G. $9\pi$ m²
- H. $36\pi$ cm²
- I. $9\pi$ cm²

---

51. Consider this true conditional statement.

If you want to buy milk, then you go to the store.

a. Write the converse and determine whether it is true or false.
b. If the converse is false, give a counterexample to show that it is false. If the converse is true, combine the original statement and its converse as a biconditional.

---

**Mixed Review**

**Write the converse of each statement.**

- 52. If you do not sleep enough, then your grades suffer.
- 53. If you are in the school chorus, then you have a good voice.

**54. Reasoning** What is the truth value of the contrapositive of a true conditional?

---

**Get Ready! To prepare for Lesson 2-4, do Exercises 55–57.**

What are the next two terms in each sequence?

- 55. 100, 90, 80, 70, . . .
- 56. 2500, 500, 100, 20, . . .
- 57. 1, 2, 0, 3, −1, . . .
Do you know HOW?

Use inductive reasoning to describe the pattern of each sequence. Then find the next two terms.

1. 1, 12, 123, 1234, . . .
2. 3, 4.5, 6.75, 10.125, . . .
3. 2, 3, 5, 7, 11, 13, . . .

Draw the next figure in each sequence.

4. [Diagram of three circles, increasing in complexity]

5. [Diagram of five shapes, increasing in complexity]

Find a counterexample for the conjecture.

6. Three coplanar lines always make a triangle.
7. All balls are spheres.
8. When it rains, it pours.

Identify the hypothesis and the conclusion of the conditional statements.

9. If the traffic light is red, then you must stop.
10. If $x > 5$, then $x^2 > 25$.
11. If you leave your house, then you must lock the door.

Rewrite the statements as conditional statements.

12. Roses are beautiful flowers.
14. Quadrilaterals have four sides.
15. The world’s largest trees are giant sequoias.

For Exercises 16–19, write the converse, inverse, and contrapositive of each conditional statement. Determine the truth value of each statement. If it is false, provide a counterexample.

16. If a figure is a circle with radius $r$, then its circumference is $2\pi r$.
17. If an integer ends with 0, then it is divisible by 2.
18. If you win the league championship game, then you win the league trophy.
19. If a triangle has one right angle, then the other two angles are complementary.

20. Write the two conditionals that make up this biconditional: An angle is an acute angle if and only if its measure is between 0 and 90.

For Exercises 21–23, rewrite the definition as a biconditional.

21. Points that lie on the same line are collinear.
22. Figures with three sides are triangles.
23. The moon is the largest satellite of Earth.

24. Which of the following is a good definition?
   - A: Grass is green.
   - B: Dinosaurs are extinct.
   - C: A pound weighs less than a kilogram.
   - D: A yard is a unit of measure exactly 3 ft long.

Do you UNDERSTAND?

25. Open-Ended Describe a situation where you used a pattern to reach a conjecture.

26. How does the word induce relate to the term inductive reasoning?

27. Error Analysis Why is the following not a good definition? How could you improve it?
   Rain is water.
Objective: To use the Law of Detachment and the Law of Syllogism

You want to use the coupon to buy three different pairs of jeans. You have narrowed your choices to four pairs. The costs of the different pairs are $24.99, $39.99, $40.99, and $50.00. If you spend as little as possible, what is the average amount per pair of jeans that you will pay? Explain.

BUY TWO PAIRS OF JEANS
Get a THIRD Free*

*Free jeans must be of equal or lesser value.

In the Solve It, you drew a conclusion based on several facts. You used deductive reasoning. Deductive reasoning (sometimes called logical reasoning) is the process of reasoning logically from given statements or facts to a conclusion.

Essential Understanding: Given true statements, you can use deductive reasoning to make a valid or true conclusion.

Take Note

<table>
<thead>
<tr>
<th>Property</th>
<th>Law of Detachment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Law</td>
<td>If the hypothesis of a true conditional is true, then the conclusion is true.</td>
</tr>
<tr>
<td>Symbols</td>
<td>If $p \rightarrow q$ is true, and $p$ is true, then $q$ is true.</td>
</tr>
</tbody>
</table>

To use the Law of Detachment, identify the hypothesis of the given true conditional. If the second given statement matches the hypothesis of the conditional, then you can make a valid conclusion.
Problem 1 Using the Law of Detachment

What can you conclude from the given true statements?

A Given: If a student gets an A on a final exam, then the student will pass the course. Felicia got an A on her history final exam.

If a student gets an A on a final exam, then the student will pass the course. Felicia got an A on her history final exam.

The second statement matches the hypothesis of the given conditional. By the Law of Detachment, you can make a conclusion.

You conclude: Felicia will pass her history course.

B Given: If a ray divides an angle into two congruent angles, then the ray is an angle bisector.

\( \overline{RS} \) divides \( \angle ARB \) so that \( \angle ARS \equiv \angle SRB \).

If a ray divides an angle into two congruent angles, then the ray is an angle bisector.

\( \overline{RS} \) divides \( \angle ARB \) so that \( \angle ARS \equiv \angle SRB \).

The second statement matches the hypothesis of the given conditional. By the Law of Detachment, you can make a conclusion.

You conclude: \( \overline{RS} \) is an angle bisector.

C Given: If two angles are adjacent, then they share a common vertex.

\( \angle 1 \) and \( \angle 2 \) share a common vertex.

If two angles are adjacent, then they share a common vertex.

\( \angle 1 \) and \( \angle 2 \) share a common vertex.

The information in the second statement about \( \angle 1 \) and \( \angle 2 \) does not tell you if the angles are adjacent. The second statement does not match the hypothesis of the given conditional, so you cannot use the Law of Detachment. \( \angle 1 \) and \( \angle 2 \) could be vertical angles, since vertical angles also share a common vertex. You cannot make a conclusion.

Got It? 1. What can you conclude from the given information?
   a. If there is lightning, then it is not safe to be out in the open.
      María sees lightning from the soccer field.
   b. If a figure is a square, then its sides have equal length.
      Figure \( ABCD \) has sides of equal length.
Another law of deductive reasoning is the Law of Syllogism. The Law of Syllogism allows you to state a conclusion from two true conditional statements when the conclusion of one statement is the hypothesis of the other statement.

**Property: Law of Syllogism**

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is true.</td>
<td>If it is July, then you are on summer vacation. If you are on summer vacation, then you work at a smoothie shop. You conclude: If it is July, then you work at a smoothie shop.</td>
</tr>
</tbody>
</table>

**Problem 2: Using the Law of Syllogism**

What can you conclude from the given information?

**A Given:** If a figure is a square, then the figure is a rectangle. If a figure is a rectangle, then the figure has four sides.

If a figure is a square, then the figure is a rectangle. If a figure is a rectangle, then the figure has four sides.

The conclusion of the first statement is the hypothesis of the second statement, so you can use the Law of Syllogism to make a conclusion.

**You conclude:** If a figure is a square, then the figure has four sides.

**B Given:** If you do gymnastics, then you are flexible. If you do ballet, then you are flexible.

If you do gymnastics, then you are flexible. If you do ballet, then you are flexible.

The statements have the same conclusion. Neither conclusion is the hypothesis of the other statement, so you cannot use the Law of Syllogism. You cannot make a conclusion.

**Got It?** 2. What can you conclude from the given information? What is your reasoning?

- **a.** If a whole number ends in 0, then it is divisible by 10. If a whole number is divisible by 10, then it is divisible by 5.
- **b.** If $AB$ and $AD$ are opposite rays, then the two rays form a straight angle. If two rays are opposite rays, then the two rays form a straight angle.
Problem 3  Using the Laws of Syllogism and Detachment

What can you conclude from the given information?

Given: If you live in Accra, then you live in Ghana.
If you live in Ghana, then you live in Africa. Aissa lives in Accra.

If you live in Accra, then you live in Ghana.
If you live in Ghana, then you live in Africa. Aissa lives in Accra.

You can use the first two statements and the Law of Syllogism to conclude:
If you live in Accra, then you live in Africa.

You can use this new conditional statement, the fact that Aissa lives in Accra, and the Law of Detachment to make a conclusion.

You conclude: Aissa lives in Africa.

Got It? 3. a. What can you conclude from the given information? What is your reasoning?
   If a river is more than 4000 mi long, then it is longer than the Amazon.
   If a river is longer than the Amazon, then it is the longest river in the world.
   The Nile is 4132 mi long.


Lesson Check

Do You Know HOW?
If possible, make a conclusion from the given true statements. What reasoning did you use?

1. If it is Tuesday, then you will go bowling.
   You go bowling.

2. If a figure is a three-sided polygon, then it is a triangle.
   Figure ABC is a three-sided polygon.

3. If it is Saturday, then you walk to work.
   If you walk to work, then you wear sneakers.

Do You UNDERSTAND?

4. Error Analysis  What is the error in the reasoning below?

Birds that weigh more than 50 pounds cannot fly. A kiwi cannot fly. So, a kiwi weighs more than 50 pounds.

5. Compare and Contrast  How is deductive reasoning different from inductive reasoning?
Practice and Problem-Solving Exercises

A Practice

If possible, use the Law of Detachment to make a conclusion. If it is not possible to make a conclusion, tell why.

6. If a doctor suspects her patient has a broken bone, then she should take an X-ray. Dr. Ngemba suspects Lily has a broken arm.
   \[ \text{Rectangle } ABCD \text{ has area } 12 \text{ cm}^2. \]

7. If a rectangle has side lengths 3 cm and 4 cm, then it has area 12 cm\(^2\).
   \[ \text{Points } X, Y, \text{ and } Z \text{ are on line } m. \]

8. If three points are on the same line, then they are collinear.
   \[ \angle XYZ \text{ is not obtuse}. \]

9. If an angle is obtuse, then it is not acute.
   \[ \text{If a student wants to go to college, then the student must study hard.} \]
   Rashid wants to go to Pennsylvania State University.

If possible, use the Law of Syllogism to make a conclusion. If it is not possible to make a conclusion, tell why.

11. Ecology If an animal is a Florida panther, then its scientific name is \textit{Puma concolor coryi}.

12. If an animal is \textit{Puma concolor coryi}, then it is endangered.

13. If a whole number ends in 6, then it is divisible by 2.

14. If a whole number ends in 4, then it is divisible by 2.

15. If a line intersects a segment at its midpoint, then the line bisects the segment.

16. If a line bisects a segment, then it divides the segment into two congruent segments.

17. If you improve your vocabulary, then you will improve your score on a standardized test.

18. If you read often, then you will improve your vocabulary.

Use the Law of Detachment and the Law of Syllogism to make conclusions from the following statements. If it is not possible to make a conclusion, tell why.

15. If a mountain is the highest in Alaska, then it is the highest in the United States.
   \[ \text{If an Alaskan mountain is more than 20,320 ft high, then it is the highest in Alaska.} \]
   \[ \text{Alaska's Mount McKinley is 20,320 ft high.} \]

16. If you live in the Bronx, then you live in New York.
   \[ \text{If you live in New York, then you live in the eleventh state to enter the Union.} \]

17. If you are studying botany, then you are studying biology.
   \[ \text{If you are studying biology, then you are studying science.} \]
   Shanti is taking science this year.
18. **Think About a Plan** If it is the night of your weekly basketball game, your family eats at your favorite restaurant. When your family eats at your favorite restaurant, you always get chicken fingers. If it is Tuesday, then it is the night of your weekly basketball game. How much do you pay for chicken fingers after your game? Use the specials board at the right to decide. Explain your reasoning.
   - How can you reorder and rewrite the sentences to help you?
   - How can you use the Law of Syllogism to answer the question?

**Beverages** For Exercises 19–24, assume that the following statements are true.

A. If Maria is drinking juice, then it is breakfast time.
B. If it is lunchtime, then Kira is drinking milk and nothing else.
C. If it is mealtine, then Curtis is drinking water and nothing else.
D. If it is breakfast time, then Julio is drinking juice and nothing else.
E. Maria is drinking juice.

Use only the information given above. For each statement, write must be true, may be true, or is not true. Explain your reasoning.

19. Julio is drinking juice.
20. Curtis is drinking water.
22. Curtis is drinking juice.
23. Maria is drinking water.
24. Julio is drinking milk.

25. **Physics** Quarks are subatomic particles identified by electric charge and rest energy. The table shows how to categorize quarks by their flavors. Show how the Law of Detachment and the table are used to identify the flavor of a quark with a charge of $-\frac{1}{3}e$ and rest energy 540 MeV.

<table>
<thead>
<tr>
<th>Rest Energy and Charge of Quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rest Energy (MeV)</td>
</tr>
<tr>
<td>Electric Charge (e)</td>
</tr>
<tr>
<td>Flavor</td>
</tr>
</tbody>
</table>

Write the first statement as a conditional. If possible, use the Law of Detachment to make a conclusion. If it is not possible to make a conclusion, tell why.

26. All national parks are interesting.
   Mammoth Cave is a national park.

27. All squares are rectangles.
   $ABCD$ is a rectangle.

28. The temperature is always above 32°F in Key West, Florida.
   The temperature is 62°F.

29. Every high school student likes art.
   Ling likes art.

30. **Writing** Give an example of a rule used in your school that could be written as a conditional. Explain how the Law of Detachment is used in applying that rule.
31. **Biology** Consider the following given statements and conclusion.

**Given:** If an animal is a fish, then it has gills.
   A turtle does not have gills.

**You conclude:** A turtle is not a fish.

a. Make a Venn diagram to illustrate the given information.
b. Use the Venn diagram to help explain why the argument uses good reasoning.

32. **Reasoning** Use the following algorithm: Choose an integer. Multiply the integer by 3. Add 6 to the product. Divide the sum by 3.

a. Complete the algorithm for four different integers. Look for a pattern in the chosen integers and in the corresponding answers. Make a conjecture that relates the chosen integers to the answers.
b. Let the variable $x$ represent the chosen integer. Apply the algorithm to $x$. Simplify the resulting expression.
c. How does your answer to part (b) confirm your conjecture in part (a)? Describe how inductive and deductive reasoning are exhibited in parts (a) and (b).

---

**Standardized Test Prep**

33. What can you conclude from the given true statements?
   If you wake up late, then you miss the bus.
   If you miss the bus, then you are late for school.
   If you are late for school, then you missed the bus.
   If you wake up late, then you are late for school.
   If you miss the bus, then you woke up late.
   If you are late for school, then you woke up late.


a. Claire is reading *Hamlet*. Who else, if anyone, must also be reading *Hamlet*?
b. Exactly three people are reading *King Lear*. Who are they? Explain.

---

**Mixed Review**

35. Write the following definition as a biconditional.
   Inductive reasoning is reasoning based on patterns you observe.

**Get Ready!** To Prepare for Lesson 2-5, do Exercises 36–39.

Use the figure at the right.

36. Name $\angle 1$ in two other ways.
37. Name $\angle 2$ in two other ways.
38. If $\angle 1 \equiv \angle 2$, name the bisector of $\angle AOC$.
39. Classify $\angle AOC$.

---

112 Chapter 2 Reasoning and Proof
Objective: To connect reasoning in algebra and geometry

Think about how each step is related to the steps before it.

Getting Ready!

Follow the steps of the brainteaser using your age. Then try it using a family member’s age. What do you notice? Explain how the brainteaser works.

- Write down your age.
- Multiply it by 10.
- Add 8 to the product.
- Double that answer and then subtract 16.
- Finally, divide the result by 2.

In the Solve It, you logically examined a series of steps. In this lesson, you will apply logical reasoning to algebraic and geometric situations.

Essential Understanding: Algebraic properties of equality are used in geometry. They will help you solve problems and justify each step you take.

In geometry, you accept postulates and properties as true. Some of the properties that you accept as true are the properties of equality from algebra.

Lesson Vocabulary
- Reflexive Property
- Symmetric Property
- Transitive Property
- proof
- two-column proof

Take note

Key Concept: Properties of Equality

Let $a$, $b$, and $c$ be any real numbers.

**Addition Property**

If $a = b$, then $a + c = b + c$.

**Subtraction Property**

If $a = b$, then $a - c = b - c$.

**Multiplication Property**

If $a = b$, then $a \cdot c = b \cdot c$.

**Division Property**

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

**Reflexive Property**

$a = a$

**Symmetric Property**

If $a = b$, then $b = a$.

**Transitive Property**

If $a = b$ and $b = c$, then $a = c$.

**Substitution Property**

If $a = b$, then $b$ can replace $a$ in any expression.
Key Concept  The Distributive Property

Use multiplication to distribute $a$ to each term of the sum or difference within the parentheses.

Sum:  
$$a(b + c) = a(b + c) = ab + ac$$

Difference:  
$$a(b - c) = a(b - c) = ab - ac$$

You use deductive reasoning when you solve an equation. You can justify each step with a postulate, a property, or a definition. For example, you can use the Distributive Property to justify combining like terms. If you think of the Distributive Property as $ab + ac = a(b + c)$ or $ab + ac = (b + c)a$, then $2x + x = (2 + 1)x = 3x$.

Problem 1  Justifying Steps When Solving an Equation

Algebra  What is the value of $x$? Justify each step.

$\angle AOM$ and $\angle MOC$ are supplementary.  Form a linear pair are supplementary.

$m\angle AOM + m\angle MOC = 180$  Definition of supplementary $\angle$  

$$(2x + 30) + x = 180$$  Substitution Property  

$3x + 30 = 180$  Distributive Property  

$3x = 150$  Subtraction Property of Equality  

$x = 50$  Division Property of Equality

Got It?  1. What is the value of $x$? Justify each step.

Given: $\overline{AB}$ bisects $\angle RAN$.

Some properties of equality have corresponding properties of congruence.

Key Concept  Properties of Congruence

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Property</td>
<td>$\overline{AB} \cong \overline{AB}$  $\angle A \cong \angle A$</td>
</tr>
<tr>
<td>Symmetric Property</td>
<td>If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.  If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.</td>
</tr>
<tr>
<td>Transitive Property</td>
<td>If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.  If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.  If $\angle B \cong \angle A$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.</td>
</tr>
</tbody>
</table>
Problem 2

Using Properties of Equality and Congruence

What is the name of the property of equality or congruence that justifies going from the first statement to the second statement?

A
\[2x + 9 = 19\]
\[2x = 10\]  
Subtraction Property of Equality

B
\[\angle O \equiv \angle W\] and \[\angle W \equiv \angle L\]
\[\angle O \equiv \angle L\]  
Transitive Property of Congruence

C
\[m\angle E = m\angle T\]
\[m\angle T = m\angle E\]  
Symmetric Property of Equality

Got It? 2. For parts (a)-(c), what is the name of the property of equality or congruence that justifies going from the first statement to the second statement?

a. \[\overline{AR} \equiv \overline{TY}\]
\[\overline{TY} \equiv \overline{AR}\]

b. \[3(x + 5) = 9\]
\[3x + 15 = 9\]

c. \[\frac{1}{4}x = 7\]
\[x = 28\]

d. Reasoning What property justifies the statement \(m\angle R = m\angle R\)?

A proof is a convincing argument that uses deductive reasoning. A proof logically shows why a conjecture is true. A two-column proof lists each statement on the left. The justification, or the reason for each statement, is on the right. Each statement must follow logically from the steps before it. The diagram below shows the setup for a two-column proof. You will find the complete proof in Problem 3.

Given: \(m\angle 1 = m\angle 3\)

Prove: \(m\angle AEC = m\angle DEB\)

The first statement is usually the given statement.

Each statement should follow logically from the previous statements.

The last statement is what you want to prove.
Problem 3 Writing a Two-Column Proof

Write a two-column proof.

Given: \( m\angle 1 = m\angle 3 \)

Prove: \( m\angle AEC = m\angle DEB \)

Know

\( m\angle 1 = m\angle 3 \)

Need

To prove that \( m\angle AEC = m\angle DEB \)

Plan

Add \( m\angle 2 \) to both \( m\angle 1 \) and \( m\angle 3 \). The resulting angles will have equal measure.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( m\angle 1 = m\angle 3 )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( m\angle 2 = m\angle 2 )</td>
<td>2) Reflexive Property of Equality</td>
</tr>
<tr>
<td>3) ( m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2 )</td>
<td>3) Addition Property of Equality</td>
</tr>
<tr>
<td>4) ( m\angle 1 + m\angle 2 = m\angle AEC )</td>
<td>4) Angle Addition Postulate</td>
</tr>
<tr>
<td>( m\angle 3 + m\angle 2 = m\angle DEB )</td>
<td></td>
</tr>
<tr>
<td>5) ( m\angle AEC = m\angle DEB )</td>
<td>5) Substitution Property</td>
</tr>
</tbody>
</table>

Got It? 3. a. Write a two-column proof.

Given: \( \overline{AB} \cong \overline{CD} \)

Prove: \( \overline{AC} \cong \overline{BD} \)

b. Reasoning In Problem 3, why is Statement 2 necessary in the proof?

Lesson Check

Do you know HOW?

Name the property of equality or congruence that justifies going from the first statement to the second statement.

1. \( m\angle A = m\angle S \) and \( m\angle S = m\angle K \)
   \( m\angle A = m\angle K \)

2. \( 3x + x + 7 = 23 \)
   \( 4x + 7 = 23 \)

3. \( 4x + 5 = 17 \)
   \( 4x = 12 \)

Do you UNDERSTAND?

4. Developing Proof Fill in the reasons for this algebraic proof.

Given: \( 5x + 1 = 21 \)

Prove: \( x = 4 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( 5x + 1 = 21 )</td>
<td>1) a. ?</td>
</tr>
<tr>
<td>2) ( 5x = 20 )</td>
<td>2) b. ?</td>
</tr>
<tr>
<td>3) ( x = 4 )</td>
<td>3) c. ?</td>
</tr>
</tbody>
</table>
Practice and Problem-Solving Exercises

Practice

Algebra Fill in the reason that justifies each step.

5. \( \frac{1}{2}x - 5 = 10 \)  Given
   \( 2\left(\frac{1}{2}x - 5\right) = 20 \)  a. ?
   \( x - 10 = 20 \)  b. ?
   \( x = 30 \)  c. ?

6. \( 5(x + 3) = -4 \)  Given
   \( 5x + 15 = -4 \)  a. ?
   \( 5x = -19 \)  b. ?
   \( x = \frac{19}{5} \)  c. ?

7. \( \angle CDE \) and \( \angle EDF \) are supplementary.
   \( m\angle CDE + m\angle EDF = 180 \)
   \( x + (3x + 20) = 180 \)
   \( 4x + 20 = 180 \)
   \( 4x = 160 \)
   \( x = 40 \)

8. \( XY = 42 \)  Given
   \( XZ + ZY = XY \)
   \( 3(n + 4) + 3n = 42 \)  a. ?
   \( 3n + 12 + 3n = 42 \)  b. ?
   \( 6n + 12 = 42 \)  c. ?
   \( 6n = 30 \)  d. ?
   \( n = 5 \)  e. ?

Name the property of equality or congruence that justifies going from the first statement to the second statement.

9. \( 2x + 1 = 7 \)
   \( 2x = 6 \)  a. ?
   \( x = 4 \)  b. ?

10. \( 5x = 20 \)
   \( x = 4 \)  c. ?

11. \( \overline{ST} \equiv \overline{QR} \)
   \( \overline{QR} \equiv \overline{ST} \)  d. ?

12. \( AB - BC = 12 \)
   \( AB = 12 + BC \)  e. ?

13. Developing Proof Fill in the missing statements or reasons for the following two-column proof.

   Given: \( C \) is the midpoint of \( \overline{AD} \).
   Prove: \( x = 6 \)

   Statements
<table>
<thead>
<tr>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( C ) is the midpoint of ( \overline{AD} ).</td>
</tr>
<tr>
<td>2) ( AC \equiv CD )</td>
</tr>
<tr>
<td>3) ( AC = CD )</td>
</tr>
<tr>
<td>4) ( 4x = 2x + 12 )</td>
</tr>
<tr>
<td>5) ( d. ? )</td>
</tr>
<tr>
<td>6) ( x = 6 )</td>
</tr>
<tr>
<td>1) a. ?</td>
</tr>
<tr>
<td>2) b. ?</td>
</tr>
<tr>
<td>3) ( \equiv ) segments have equal length.</td>
</tr>
<tr>
<td>4) c. ?</td>
</tr>
<tr>
<td>5) Subtraction Property of Equality</td>
</tr>
<tr>
<td>6) e. ?</td>
</tr>
</tbody>
</table>
14. Symmetric Property of Equality
   If \( AB = YU \), then \( ? \).
15. Symmetric Property of Congruence
   If \( \angle H \cong \angle K \), then \( ? \) = \( \angle H \).
16. Reflexive Property of Congruence
   \( \angle POR \equiv \angle ? \).
17. Distributive Property
   \( 3(x - 1) = 3x - ? \).
18. Substitution Property
   If \( LM = 7 \) and \( EF + LM = NP \), then \( ? = NP \).
19. Transitive Property of Congruence
   If \( \angle XYZ \cong \angle AOB \) and
   \( \angle AOB \cong \angle WYT \), then \( ? \).

20. Think About a Plan  A very important part in writing proofs is analyzing
   the diagram for key information. What true statements can you make
   based on the diagram at the right?
   - What theorems or definitions relate to the geometric figures
     in the diagram?
   - What types of markings show relationships between parts
     of geometric figures?

21. Writing  Explain why the statements \( \overline{LR} \cong \overline{RL} \) and \( \angle CBA \cong \angle ABC \) are both true
   by the Reflexive Property of Congruence.

22. Reasoning  Complete the following statement. Describe the reasoning that
   supports your answer.
   The Transitive Property of Falling Dominoes: If Domino A causes Domino B to fall,
   and Domino B causes Domino C to fall, then Domino A causes Domino \( ? \) to fall.

23. Given: \( KM = 35 \)
   Prove: \( KL = 15 \)

24. Given: \( m \angle GFI = 128 \)
   Prove: \( m \angle EFI = 40 \)

25. Error Analysis  The steps below “show” that \( 1 = 2 \). Describe the error.
   \[ a = b \quad \text{Given} \]
   \[ ab = b^2 \quad \text{Multiplication Property of Equality} \]
   \[ ab - a^2 = b^2 - a^2 \quad \text{Subtraction Property of Equality} \]
   \[ a(b - a) = (b + a)(b - a) \quad \text{Distributive Property} \]
   \[ a = b + a \quad \text{Division Property of Equality} \]
   \[ a = a + a \quad \text{Substitution Property} \]
   \[ a = 2a \quad \text{Simplify} \]
   \[ 1 = 2 \quad \text{Division Property of Equality} \]
Relationships  Consider the following relationships among people. Tell whether each relationship is reflexive, symmetric, transitive, or none of these. Explain.

Sample:  The relationship “is younger than” is not reflexive because Sue is not younger than herself. It is not symmetric because if Sue is younger than Fred, then Fred is not younger than Sue. It is transitive because if Sue is younger than Fred and Fred is younger than Alana, then Sue is younger than Alana.

26. has the same birthday as  27. is taller than  28. lives in a different state than

Standardized Test Prep

29. You are typing a one-page essay for your English class. You set 1-in. margins on all sides of the page as shown in the figure at the right. How many square inches of the page will contain your essay?

30. Given $2(m\angle A) + 17 = 45$ and $m\angle B = 2(m\angle A)$, what is $m\angle B$?

31. A circular flowerbed has circumference $14\pi$ m. What is its area in square meters? Use $3.14$ for $\pi$.

32. The measure of the supplement of $\angle 1$ is 98. What is $m\angle 1$?

33. What is the next term in the sequence 2, 4, 8, 14, 22, 32, 44, …?

Mixed Review

34. Reasoning  Use logical reasoning to draw a conclusion.

If a student is having difficulty in class, then that student’s teacher is concerned.

Walt is having difficulty in science class.

Use the diagram at the right. Find each measure.

35. $m\angle AOC$  36. $m\angle DOB$
37. $m\angle AOD$  38. $m\angle BOE$

Get Ready!  To prepare for Lesson 2-6, do Exercises 39–41.

Find the value of each variable.

39.  40.  41.
Proving Angles Congruent

**Objective**
To prove and apply theorems about angles

**Lesson Vocabulary**
- theorem
- paragraph proof

**Solve It**
A quilter wants to duplicate this quilt but knows the measure of only two angles. What are the measures of angles 1, 2, 3, and 4? How do you know?

Use what you've learned about congruent angle pairs.

In the Solve It, you may have noticed a relationship between vertical angles. You can prove that this relationship is always true using deductive reasoning. A **theorem** is a conjecture or statement that you prove true.

**Essential Understanding**
You can use given information, definitions, properties, postulates, and previously proven theorems as reasons in a proof.

**Take Note**
**Theorem 2-1** **Vertical Angles Theorem**

Vertical angles are congruent.

\[ \angle 1 \cong \angle 3 \text{ and } \angle 2 \cong \angle 4 \]

When you are writing a geometric proof, it may help to separate the theorem you want to prove into a **hypothesis** and **conclusion**. Another way to write the Vertical Angles Theorem is “If two angles are vertical, then they are congruent.” The hypothesis becomes the given statement, and the conclusion becomes what you want to prove. A two-column proof of the Vertical Angles Theorem follows.

120  Chapter 2  Reasoning and Proof
**Proof of Theorem 2-1: Vertical Angles Theorem**

**Given:** \( \angle 1 \) and \( \angle 3 \) are vertical angles.

**Prove:** \( \angle 1 \equiv \angle 3 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \angle 1 ) and ( \angle 3 ) are vertical angles.</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle 1 ) and ( \angle 2 ) are supplementary. ( \angle 2 ) and ( \angle 3 ) are supplementary.</td>
<td>2) ( \angle )s that form a linear pair are supplementary.</td>
</tr>
<tr>
<td>3) ( m\angle 1 + m\angle 2 = 180 ) ( m\angle 2 + m\angle 3 = 180 )</td>
<td>3) The sum of the measures of supplementary ( \angle )s is 180.</td>
</tr>
<tr>
<td>4) ( m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3 )</td>
<td>4) Transitive Property of Equality</td>
</tr>
<tr>
<td>5) ( m\angle 1 = m\angle 3 )</td>
<td>5) Subtraction Property of Equality</td>
</tr>
<tr>
<td>6) ( \angle 1 \equiv \angle 3 )</td>
<td>6) ( \angle )s with the same measure are ( \equiv ).</td>
</tr>
</tbody>
</table>

**Problem 1**

Using the Vertical Angles Theorem

**Think**

The two labeled angles are vertical angles, so set them equal.

Solve for \( x \) by subtracting 2\( x \) from each side and then dividing by 2.

Grid the answer as 21/2 or 10.5.

**Write**

\[ 2x + 21 = 4x \]

\[ 21 = 2x \]

\[ \frac{21}{2} = x \]

**Got It?**

1. What is the value of \( x \)?

121
Think

Why does the Transitive Property work for statements 3 and 5?
In each case, an angle is congruent to two other angles, so the two angles are congruent to each other.

**Problem 2** Proof Using the Vertical Angles Theorem

**Given:** \( \angle 1 \equiv \angle 4 \)
**Prove:** \( \angle 2 \equiv \angle 3 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \angle 1 \equiv \angle 4 )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle 4 \equiv \angle 2 )</td>
<td>2) Vertical angles are ( \equiv )</td>
</tr>
<tr>
<td>3) ( \angle 1 \equiv \angle 2 )</td>
<td>3) Transitive Property of Congruence</td>
</tr>
<tr>
<td>4) ( \angle 1 \equiv \angle 3 )</td>
<td>4) Vertical angles are ( \equiv )</td>
</tr>
<tr>
<td>5) ( \angle 2 \equiv \angle 3 )</td>
<td>5) Transitive Property of Congruence</td>
</tr>
</tbody>
</table>

**Got It?**

2. a. Use the Vertical Angles Theorem to prove the following.

**Given:** \( \angle 1 \equiv \angle 2 \)
**Prove:** \( \angle 1 \equiv \angle 2 \equiv \angle 3 \equiv \angle 4 \)

b. **Reasoning** How can you prove \( \angle 1 \equiv \angle 2 \equiv \angle 3 \equiv \angle 4 \) without using the Vertical Angles Theorem? Explain.

The proof in Problem 2 is two-column, but there are many ways to display a proof. A paragraph proof is written as sentences in a paragraph. Below is the proof from Problem 2 in paragraph form. Each statement in the Problem 2 proof is red in the paragraph proof.

**Proof**

**Given:** \( \angle 1 \equiv \angle 4 \)
**Prove:** \( \angle 2 \equiv \angle 3 \)

**Proof:** \( \angle 1 \equiv \angle 4 \) is given. \( \angle 4 \equiv \angle 2 \) because vertical angles are congruent. By the Transitive Property of Congruence, \( \angle 1 \equiv \angle 2 \), \( \angle 1 \equiv \angle 3 \) because vertical angles are congruent. By the Transitive Property of Congruence, \( \angle 2 \equiv \angle 3 \).

The Vertical Angles Theorem is a special case of the following theorem.

**Theorem 2-2** Congruent Supplements Theorem

**Theorem**
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

**If . . .**
\( \angle 1 \) and \( \angle 3 \) are supplements and \( \angle 2 \) and \( \angle 3 \) are supplements.

**Then . . .**
\( \angle 1 \equiv \angle 2 \)

You will prove Theorem 2-2 in Problem 3.
**Problem 3** Writing a Paragraph Proof

**Given:** \( \angle 1 \) and \( \angle 3 \) are supplementary.
\( \angle 2 \) and \( \angle 3 \) are supplementary.

**Prove:** \( \angle 1 \cong \angle 2 \)

**Proof:** \( \angle 1 \) and \( \angle 3 \) are supplementary because it is given. So \( m\angle 1 + m\angle 3 = 180 \) by the definition of supplementary angles. \( \angle 2 \) and \( \angle 3 \) are supplementary because it is given, so \( m\angle 2 + m\angle 3 = 180 \) by the same definition. By the Transitive Property of Equality, \( m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3 \). Subtract \( m\angle 3 \) from each side. By the Subtraction Property of Equality, \( m\angle 1 = m\angle 2 \). Angles with the same measure are congruent, so \( \angle 1 \cong \angle 2 \).

---

**Got It?** 3. Write a paragraph proof for the Vertical Angles Theorem.

The following theorems are similar to the Congruent Supplements Theorem.

### Theorem 2-3 Congruent Complements Theorem

**Theorem**
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

**If . . .**
\( \angle 1 \) and \( \angle 2 \) are complements and \( \angle 3 \) and \( \angle 2 \) are complements

**Then . . .**
\( \angle 1 \cong \angle 3 \)

You will prove Theorem 2-3 in Exercise 13.

### Theorem 2-4

**Theorem**
All right angles are congruent.

**If . . .**
\( \angle 1 \) and \( \angle 2 \) are right angles

**Then . . .**
\( \angle 1 \cong \angle 2 \)

You will prove Theorem 2-4 in Exercise 18.

### Theorem 2-5

**Theorem**
If two angles are congruent and supplementary, then each is a right angle.

**If . . .**
\( \angle 1 \cong \angle 2 \), and \( \angle 1 \) and \( \angle 2 \) are supplements

**Then . . .**
\( m\angle 1 = m\angle 2 = 90 \)

You will prove Theorem 2-5 in Exercise 23.
Lesson Check

Do you know HOW?
1. What are the measures of \( \angle 1 \), \( \angle 2 \), and \( \angle 3 \)?

![Diagram with angles labeled: \( \angle 1 \) at 40°, \( \angle 2 \) at 50°, and \( \angle 3 \) at 130°.]

2. What is the value of \( x \)?

- A 12
- E 20
- C 120
- D 136

Do you UNDERSTAND?

3. **Reasoning** If \( \angle A \) and \( \angle B \) are supplements, and \( \angle A \) and \( \angle C \) are supplements, what can you conclude about \( \angle B \) and \( \angle C \)? Explain.

4. **Error Analysis** Your friend knows that \( \angle 1 \) and \( \angle 2 \) are complementary and that \( \angle 1 \) and \( \angle 3 \) are complementary. He concludes that \( \angle 2 \) and \( \angle 3 \) must be complementary. What is his error in reasoning?

5. **Compare and Contrast** How is a theorem different from a postulate?

Practice and Problem-Solving Exercises

**Practice**

Find the value of each variable.

6. \( 3x \) \( \square \) \( 80 - x \)°

7. \( 2x \) \( \square \) 76°

8. \( (x + 90)° \) \( \square \) 4x°

Find the measures of the labeled angles in each exercise.

9. Exercise 6

10. Exercise 7

11. Exercise 8

12. **Developing Proof** Complete the following proof by filling in the blanks.

**Given:** \( \angle 1 \cong \angle 3 \)

**Prove:** \( \angle 6 \cong \angle 4 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \angle 1 \cong \angle 3 )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle 3 \cong \angle 6 )</td>
<td>2) a. ?</td>
</tr>
<tr>
<td>3) ( b. ? )</td>
<td>3) Transitive Property of Congruence</td>
</tr>
<tr>
<td>4) ( \angle 1 \cong \angle 4 )</td>
<td>4) c. ?</td>
</tr>
<tr>
<td>5) ( \angle 6 \cong \angle 4 )</td>
<td>5) d. ?</td>
</tr>
</tbody>
</table>
13. Developing Proof Fill in the blanks to complete this proof of the Congruent Complements Theorem (Theorem 2-3).

If two angles are complements of the same angle, then the two angles are congruent.

**Given:** \( \angle 1 \) and \( \angle 2 \) are complementary.
\( \angle 3 \) and \( \angle 2 \) are complementary.

**Prove:** \( \angle 1 \cong \angle 3 \)

**Proof:** \( \angle 1 \) and \( \angle 2 \) are complementary and \( \angle 3 \) and \( \angle 2 \) are complementary because it is given. By the definition of complementary angles,
\[ m\angle 1 + m\angle 2 = a. \Box \] and \( m\angle 3 + m\angle 2 = b. \Box \). Then
\[ m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2 \] by the Transitive Property of Equality. Subtract \( m\angle 2 \) from each side. By the Subtraction Property of Equality, you get \( m\angle 1 = c. \Box \). Angles with the same measure are \( d. \Box \), so \( \angle 1 \cong \angle 3 \).

14. Think About a Plan What is the measure of the angle formed by Park St. and 118th St.?
- Can you make a connection between the angle you need to find and the labeled angle?
- How are angles that form a right angle related?

15. Open-Ended Give an example of vertical angles in your home or classroom.

**Algebra** Find the value of each variable and the measure of each labeled angle.

16. \( (x + 10)^\circ \) \( (4x - 35)^\circ \)

17. \( (3x + 8)^\circ \) \( (5x - 20)^\circ \) \( (5x + 4y) \)

18. Developing Proof Fill in the blanks to complete this proof of Theorem 2-4.

All right angles are congruent.

**Given:** \( \angle X \) and \( \angle Y \) are right angles.

**Prove:** \( \angle X \cong \angle Y \)

**Proof:** \( \angle X \) and \( a. \Box \) are right angles because it is given.
By the definition of \( b. \Box \), \( m\angle X = 90 \) and \( m\angle Y = 90 \).
By the Transitive Property of Equality, \( m\angle X = c. \Box \).
Because angles of equal measure are congruent, \( d. \Box \).

19. Miniature Golf In the game of miniature golf, the ball bounces off the wall at the same angle it hit the wall. (This is the angle formed by the path of the ball and the line perpendicular to the wall at the point of contact.) In the diagram, the ball hits the wall at a 40° angle. Using Theorem 2-3, what are the values of \( x \) and \( y \)?
Name two pairs of congruent angles in each figure. Justify your answers.

20. [Diagram]

21. [Diagram]

22. [Diagram]

23. **Developing Proof** Fill in the blanks to complete this proof of Theorem 2-5.

If two angles are congruent and supplementary, then each is a right angle.

**Given:** \( \angle W \) and \( \angle V \) are congruent and supplementary.

**Prove:** \( \angle W \) and \( \angle V \) are right angles.

**Proof:** \( \angle W \) and \( \angle V \) are congruent because \( \text{a. } \) \( \text{b. } \) \( \text{c. } \) \( \text{d. } \) \( \text{e. } \)

Because congruent angles have the same measure, \( m \angle W = \) \( m \angle V \).

\( \angle W \) and \( \angle V \) are supplementary because it is given. By the definition of supplementary angles, \( m \angle W + m \angle V = \) \( \text{c. } \) \( \text{d. } \) \( \text{e. } \) \( \text{f. } \) \( \text{g. } \).

Substituting \( m \angle W \) for \( m \angle V \), you get \( m \angle W + m \angle W = 180 \), or \( 2m \angle W = 180 \). By the \( \text{d. } \) \( \text{e. } \) Property of Equality, \( m \angle W = 90 \).

Since \( m \angle W = m \angle V \), \( m \angle V = 90 \) by the Transitive Property of Equality. Both angles are \( \text{e. } \) \( \text{f. } \) angles by the definition of right angles.

24. **Design** In the photograph, the legs of the table are constructed so that \( \angle 1 \equiv \angle 2 \). What theorem can you use to justify the statement that \( \angle 3 \equiv \angle 4 \)?

25. **Reasoning** Explain why this statement is true: If \( m \angle ABC + m \angle XYZ = 180 \) and \( \angle ABC \equiv \angle XYZ \), then \( \angle ABC \) and \( \angle XYZ \) are right angles.

**Algebra** Find the measure of each angle.

26. \( \angle A \) is twice as large as its complement, \( \angle B \).

27. \( \angle A \) is half as large as its complement, \( \angle B \).

28. \( \angle A \) is twice as large as its supplement, \( \angle B \).

29. \( \angle A \) is half as large as twice its supplement, \( \angle B \).

30. Write a proof for this form of Theorem 2-2.

**Proof**

If two angles are supplements of congruent angles, then the two angles are congruent.

**Given:** \( \angle 1 \) and \( \angle 2 \) are supplementary.

\( \angle 3 \) and \( \angle 4 \) are supplementary.

\( \angle 2 \equiv \angle 4 \)

**Prove:** \( \angle 1 \equiv \angle 3 \)

31. **Coordinate Geometry** \( \angle DOE \) contains points \( D(2, 3) \), \( O(0, 0) \), and \( E(5, 1) \). Find the coordinates of a point \( F \) so that \( \overrightarrow{OF} \) is a side of an angle that is adjacent and supplementary to \( \angle DOE \).
32. **Coordinate Geometry** \( \angle AOX \) contains points \( A(1, 3) \), \( O(0, 0) \), and \( X(4, 0) \).
   a. Find the coordinates of a point \( B \) so that \( \angle BOA \) and \( \angle AOX \) are adjacent complementary angles.
   b. Find the coordinates of a point \( C \) so that \( \overline{OC} \) is a side of a different angle that is adjacent and complementary to \( \angle AOX \).

**Algebra** Find the value of each variable and the measure of each angle.

33. \( 2x^\circ \) \( (x + y)^\circ \) \( (y - x)^\circ \)

34. \( y^\circ \) \( 2x^\circ \) \( (x + y + 5)^\circ \)

35. \( 4y^\circ \) \( 2x^\circ \) \( (x + y + 10)^\circ \)

---

**Standardized Test Prep**

36. \( \angle 1 \) and \( \angle 2 \) are vertical angles. If \( m\angle 1 = 63 \) and \( m\angle 2 = 4x - 9 \), what is the value of \( x \)?

37. What is the area in square centimeters of a triangle with a base of 5 cm and a height of 8 cm?

38. In the figure at the right, \( m\angle 1 = \frac{1}{2}(m\angle 2) \), \( m\angle 2 = \frac{2}{3}(m\angle 3) \). If \( m\angle 3 = 72 \), what is \( m\angle A \)?

39. What is the measure of an angle with a supplement that is four times its complement?

---

**Mixed Review**

Which property of equality or congruence justifies going from the first statement to the second?

40. \( 3x + 7 = 19 \)

41. \( 4x = 20 \)

42. \( \angle 1 \equiv \angle 2 \) and \( \angle 3 \equiv \angle 2 \)

43. \( \angle 1 \equiv \angle 3 \)

**Get Ready!** To prepare for Lesson 3-1, do Exercises 43–48.

Refer to the figure at the right.

43. Name four points on line \( t \).

44. Are points \( G \), \( A \), and \( B \) collinear?

45. Are points \( F \), \( I \), and \( H \) collinear?

46. Name the line on which point \( E \) lies.

47. Name line \( t \) in three other ways.

48. Name the point at which lines \( t \) and \( r \) intersect.
**BIG idea** Reasoning and Proof

You can observe patterns to make a conjecture; you can prove a conjecture is true by using given information, definitions, properties, postulates, and theorems.

**Task 1**

You have the yellow game piece, your friend has the red game piece, and your brother has the blue game piece. Read the rules of the board game and then answer the questions.

**Rules**

- You play counterclockwise.
- If you land on red, then you go back 1.
- If you land on green, then you advance 1.
- If you land on yellow, then you pick a card.
  
  a. You roll 3. What must you do next? How do you know?
  
  b. Your brother picks a card at the end of his turn.
     
     On what colors might he have landed? Explain.
  
  c. Your friend rolls 2. What else must your friend do? How do you know?
  
  d. Based on the colors already shown on the board, what color should the roll-again box be? Justify your answer.

**Task 2**

Consider the number pattern at the right.

a. What is the sum of the numbers 31–40?

b. What is the sum of the numbers 101–110?

c. What kind of reasoning did you use in parts (a) and (b)?

d. Following is the development of a formula for the sum of \( n \) consecutive integers.

\[
\begin{align*}
S &= x + (x + 1) + (x + 2) + \ldots + (y - 2) + (y - 1) + y \\
2S &= (x + y) + (x + y) + (x + y) + \ldots + (x + y) + (x + y) + (x + y)
\end{align*}
\]

Add the equations.

The sum of \( n \) integers from \( x \) to \( y \): \( S = \frac{n(x + y)}{2} \)

The sum of \( n \) integers in reverse order: \( S = \frac{2S - n(x + y)}{2} \)

The sum of \( n \) terms of \( (x + y) \): \( S = \frac{n(x + y)}{2} \)

Divide each side by 2.

There are \( n \) terms of \( (x + y) \).

Use the formula to find the sum of the numbers 101–110.

e. What kind of reasoning did you use in part (d)?
Chapter Vocabulary

- biconditional (p. 98)
- conclusion (p. 89)
- conditional (p. 89)
- conjecture (p. 83)
- contrapositive (p. 91)
- converse (p. 91)
- counterexample (p.84)
- deductive reasoning (p. 106)
- equivalent statements (p. 91)
- hypothesis (p. 89)
- inductive reasoning (p. 82)
- inverse (p. 91)
- Law of Detachment (p. 106)
- Law of Syllogism (p. 108)
- negation (p. 91)
- paragraph proof (p. 122)
- proof (p. 115)
- theorem (p. 120)
- truth value (p. 90)
- two-column proof (p. 115)

Choose the correct vocabulary term to complete each sentence.

1. The part of a conditional that follows “then” is the ___.
2. Reasoning logically from given statements to a conclusion is ___.
3. A conditional has a(n) ___ of true or false.
4. The ___ of a conditional switches the hypothesis and conclusion.
5. When a conditional and its converse are true, you can write them as a single true statement called a(n) ___.
6. A statement that you prove true is a(n) ___.
7. The part of a conditional that follows “if” is the ___.
2-1 Patterns and Inductive Reasoning

Quick Review
You use **inductive reasoning** when you make conclusions based on patterns you observe. A **conjecture** is a conclusion you reach using inductive reasoning. A **counterexample** is an example that shows a conjecture is incorrect.

Example
Describe the pattern. What are the next two terms in the sequence?

1, −3, 9, −27, ...

Each term is −3 times the previous term. The next two terms are $-27 \times (-3) = 81$ and $81 \times (-3) = -243$.

Exercises
Find a pattern for each sequence. Describe the pattern and use it to show the next two terms.

8. 1000, 100, 10, ...
9. 5, −5, 5, −5, ...
10. 34, 27, 20, 13, ...
11. 6, 24, 96, 384, ...

Find a counterexample to show that each conjecture is false.

12. The product of any integer and 2 is greater than 2.
13. The city of Portland is in Oregon.

2-2 Conditional Statements

Quick Review
A **conditional** is an *if-then* statement. The symbolic form of a conditional is $p \rightarrow q$, where $p$ is the **hypothesis** and $q$ is the **conclusion**.

- To find the **converse**, switch the hypothesis and conclusion of the conditional ($q \rightarrow p$).
- To find the **inverse**, negate the hypothesis and the conclusion of the conditional ($\sim p \rightarrow \sim q$).
- To find the **contrapositive**, negate the hypothesis and the conclusion of the converse ($\sim q \rightarrow \sim p$).

Example
What is the converse of the conditional statement below?

*If you are a teenager, then you are younger than 20.*

Converse: *If you are younger than 20, then you are a teenager.*

A 7-year-old is not a teenager. The converse is false.

Exercises
Rewrite each sentence as a conditional statement.

14. All motorcyclists wear helmets.
15. Two nonparallel lines intersect in one point.
16. Angles that form a linear pair are supplementary.
17. School is closed on certain holidays.

Write the converse, inverse, and contrapositive of the given conditional. Then determine the truth value of each statement.

18. If an angle is obtuse, then its measure is greater than 90 and less than 180.
19. If a figure is a square, then it has four sides.
20. If you play the tuba, then you play an instrument.
21. If you baby-sit, then you are busy on Saturday night.
2-3 Biconditionals and Definitions

Quick Review
When a conditional and its converse are true, you can combine them as a true biconditional using the phrase if and only if. The symbolic form of a biconditional is \( p \leftrightarrow q \). You can write a good definition as a true biconditional.

Example
Is the following definition reversible? If yes, write it as a true biconditional.
A hexagon is a polygon with exactly six sides.
Yes. The conditional is true: If a figure is a hexagon, then it is a polygon with exactly six sides. Its converse is also true: If a figure is a polygon with exactly six sides, then it is a hexagon.
Biconditional: A figure is a hexagon if and only if it is a polygon with exactly six sides.

Exercises
Determine whether each statement is a good definition. If not, explain.
22. A newspaper has articles you read.
23. A linear pair is a pair of adjacent angles whose noncommon sides are opposite rays.
24. An angle is a geometric figure.
25. Write the following definition as a biconditional.
An oxymoron is a phrase that contains contradictory terms.
26. Write the following biconditional as two statements, a conditional and its converse.
Two angles are complementary if and only if the sum of their measures is 90.

2-4 Deductive Reasoning

Quick Review
Deductive reasoning is the process of reasoning logically from given statements to a conclusion.
Law of Detachment: If \( p \rightarrow q \) is true and \( p \) is true, then \( q \) is true.
Law of Syllogism: If \( p \rightarrow q \) and \( q \rightarrow r \) are true, then \( p \rightarrow r \) is true.

Example
What can you conclude from the given information?
Given: If you play hockey, then you are on the team.
If you are on the team, then you are a varsity athlete.
The conclusion of the first statement matches the hypothesis of the second statement. Use the Law of Syllogism to conclude: If you play hockey, then you are a varsity athlete.

Exercises
Use the Law of Detachment to make a conclusion.
27. If you practice tennis every day, then you will become a better player. Colin practices tennis every day.
28. \( \angle 1 \) and \( \angle 2 \) are supplementary. If two angles are supplementary, then the sum of their measures is 180.
Use the Law of Syllogism to make a conclusion.
29. If two angles are vertical, then they are congruent. If two angles are congruent, then their measures are equal.
30. If your father buys new gardening gloves, then he will work in his garden. If he works in his garden, then he will plant tomatoes.
Quick Review
You use deductive reasoning and properties to solve equations and justify your reasoning.

A proof is a convincing argument that uses deductive reasoning. A two-column proof lists each statement on the left and the justification for each statement on the right.

Example
What is the name of the property that justifies going from the first line to the second line?

\[ \angle A \equiv \angle B \text{ and } \angle B \equiv \angle C \]

\[ \angle A \equiv \angle C \]

Transitive Property of Congruence

Exercises
31. Algebra Fill in the reason that justifies each step.

Given: \( QS = 42 \)

Prove: \( x = 13 \)

\[
\begin{array}{c}
x + 3 \quad 2x \\
\quad R \quad S
\end{array}
\]

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( QS = 42 )</td>
<td>1) a. (?)</td>
</tr>
<tr>
<td>2) ( QR + RS = QS )</td>
<td>2) b. (?)</td>
</tr>
<tr>
<td>3) ( (x + 3) + 2x = 42 )</td>
<td>3) c. (?)</td>
</tr>
<tr>
<td>4) ( 3x + 3 = 42 )</td>
<td>4) d. (?)</td>
</tr>
<tr>
<td>5) ( 3x = 39 )</td>
<td>5) e. (?)</td>
</tr>
<tr>
<td>6) ( x = 13 )</td>
<td>6) f. (?)</td>
</tr>
</tbody>
</table>

Use the given property to complete the statement.

32. Division Property of Equality
If \( 2(AX) = 2(BY) \), then \( AX = ? \).

33. Distributive Property: \( 3p - 6q = 3( ? ) \)

2-6 Proving Angles Congruent

Quick Review
A statement that you prove true is a theorem. A proof written as a paragraph is a paragraph proof. In geometry, each statement in a proof is justified by given information, a property, postulate, definition, or theorem.

Example
Write a paragraph proof.

Given: \( \angle 1 \equiv \angle 4 \)

Prove: \( \angle 2 \equiv \angle 3 \)

\( \angle 1 \equiv \angle 4 \) because it is given. \( \angle 1 \equiv \angle 2 \) because vertical angles are congruent. \( \angle 4 \equiv \angle 2 \) by the Transitive Property of Congruence. \( \angle 4 \equiv \angle 3 \) because vertical angles are congruent. \( \angle 2 \equiv \angle 3 \) by the Transitive Property of Congruence.

Exercises
Use the diagram for Exercises 34–37.

34. Find the value of \( y \).

35. Find \( m\angle AEC \).

36. Find \( m\angle BED \).

37. Find \( m\angle AEB \).

38. Given: \( \angle 1 \) and \( \angle 2 \) are complementary. \( \angle 3 \) and \( \angle 4 \) are complementary. \( \angle 2 \equiv \angle 4 \)

Prove: \( \angle 1 \equiv \angle 3 \)
Do you know **HOW?**
Use inductive reasoning to describe each pattern and find the next two terms of each sequence.

1. \(-16, 8, -4, 2, \ldots\)
2. \(1, 4, 9, 16, 25, \ldots\)

For Exercises 3 and 4, find a counterexample.

3. All snakes are poisonous.
4. If two angles are complementary, then they are not congruent.

5. Identify the hypothesis and conclusion:
   If \(x + 9 = 11\), then \(x = 2\).
6. Write “all puppies are cute” as a conditional.

Write the converse, inverse, and contrapositive for each statement. Determine the truth value of each.

7. If a figure is a square, then it has at least two right angles.
8. If a square has side length 3 m, then its perimeter is 12 m.

**Writing** Explain why each statement is not a good definition.

9. A pen is a writing instrument.
10. Supplementary angles are angles that form a straight line.
11. Vertical angles are angles that are congruent.

Name the property that justifies each statement.

12. If \(UV = KL\) and \(KL = 6\), then \(UV = 6\).
13. If \(m\angle 1 + m\angle 2 = m\angle 4 + m\angle 2\), then \(m\angle 1 = m\angle 4\).
14. \(\angle ABC \equiv \angle ABC\)
15. If \(\angle DEF \equiv \angle HJK\), then \(\angle HJK \equiv \angle DEF\).
16. The measure of an angle is 52 more than the measure of its complement. What is the measure of the angle?
17. Rewrite this biconditional as two conditionals.
   A fish is a bluegill if and only if it is a bluish, freshwater sunfish.

For each diagram, state two pairs of angles that are congruent. Justify your answers.

18. \(\angle L, \angle P\)
19. \(\angle E, \angle F\)

Use the Law of Detachment and the Law of Syllogism to make any possible conclusion. Write not possible if you cannot make any possible conclusion.

20. People who live in glass houses should not throw stones. Emily should not throw stones.
21. James wants to be a chemical engineer. If a student wants to be a chemical engineer, then that student must graduate from college.

Do you **UNDERSTAND?**

22. **Open-Ended** Write two different sequences whose first three terms are 1, 2, 4. Describe each pattern.

23. **Developing Proof** Complete this proof by filling in the blanks.
   **Given:** \(\angle FED\) and \(\angle DEW\) are complementary.
   **Prove:** \(\angle FEW\) is a right angle.

\(\angle FED\) and \(\angle DEW\) are complementary because it is given. By the Definition of Complementary Angles, \(m\angle FED + m\angle DEW = \text{a. } 90\).
\(m\angle FED + m\angle DEW = m\angle FEW\) by the \(\text{b. } 90 = m\angle FEW\) by the \(\text{c. } \) Property of Equality. Then \(\angle FEW\) is a right angle by the \(\text{d. } \).
Some questions ask you to extend a pattern. Read the sample question at the right. Then follow the tips to answer it.

TIP 1
Look for a relationship between consecutive figures. Make sure the relationship holds for each pair of consecutive figures, not just the first two figures.

TIP 2
Use the relationship between the figures to extend the pattern.

Think It Through
The second figure has 2 more dots than the first figure, the third figure has 3 more dots than the second figure, and the fourth figure has 4 more dots than the third figure. So, the fifth figure will have 5 more dots than the fourth figure, or $10 + 5 = 15$ dots. The sixth figure will have 6 more dots than the fifth figure, or $15 + 6 = 21$ dots. The correct answer is D.

Vocabulary Builder
As you solve test items, you must understand the meanings of mathematical terms. Choose the correct term to complete each sentence.

I. Reasoning that is based on patterns you observe is called (inductive, deductive) reasoning.

II. The (Law of Sylogism, Law of Detachment) allows you to state a conclusion from two true conditional statements when the conclusion of one statement is the hypothesis of the other statement.

III. A conditional, or if-then, statement has two parts. The part following if is the (conclusion, hypothesis).

IV. The (Reflexive Property, Symmetric Property) says that if $a = b$, then $b = a$.

V. The (inverse, converse) of a conditional negates both the hypothesis and the conclusion.

Multiple Choice
Read each question. Then write the letter of the correct answer on your paper.

1. Which pair of angles must be congruent?
   A. supplementary angles
   B. complementary angles
   C. adjacent angles
   D. vertical angles

2. Which of the following best defines a postulate?
   E. a statement accepted without proof
   G. a conclusion reached using inductive reasoning
   H. an example that proves a conjecture false
   I. a statement that you prove true
3. What is the second step in constructing \( \angle S \), an angle congruent to \( \angle A \)?

![Diagram of angle construction](image)

4. What is the converse of the following statement?
   If a whole number has 0 as its last digit, then the number is evenly divisible by 10.
   - A. If a number is evenly divisible by 10, then it is a whole number.
   - B. If a whole number is divisible by 10, then it is an even number.
   - C. If a whole number is evenly divisible by 10, then it has 0 as its last digit.
   - D. If a whole number has 0 as its last digit, then it must be evenly divisible by 10.

5. The sum of the measures of the complement and the supplement of an angle is 114. What is the measure of the angle?
   - A. 12
   - B. 66
   - C. 78
   - D. 102

6. Which counterexample shows that the following conjecture is false?
   Every perfect square number has exactly three factors.
   - F. The factors of 2 are 1, 2.
   - G. The factors of 4 are 1, 2, 4.
   - H. The factors of 8 are 1, 2, 4, 8.
   - I. The factors of 16 are 1, 2, 4, 8, 16.

7. How many rays are in the next two terms in the sequence?

   ![Diagram of ray sequence](image)
   - A. 16 and 33 rays
   - B. 17 and 34 rays
   - C. 17 and 33 rays
   - D. 18 and 34 rays

8. Which of the statements could be a conclusion based on the following information?
   If a polygon is a pentagon, then it has one more side than a quadrilateral. If a polygon has one more side than a quadrilateral, then it has two more sides than a triangle.
   - F. If a polygon is a pentagon, then it has many sides.
   - G. If a polygon has two more sides than a quadrilateral, then it is a hexagon.
   - H. If a polygon has more sides than a triangle, then it is a pentagon.
   - I. If a polygon is a pentagon, then it has two more sides than a triangle.

9. The radius of each of the circular sections in the dumbbell-shaped table below is 3 ft. The rectangular portion has an area of 32 ft\(^2\) and the length is twice the width. What is the area of the entire table to the nearest tenth?

   ![Diagram of table](image)
   - A. 88.5 ft\(^2\)
   - B. 124.5 ft\(^2\)
   - C. 132.5 ft\(^2\)
   - D. 166.5 ft\(^2\)
10. Which of the following statements does NOT have a counterexample?

- Every month has at least 30 days.
- The product of two fractions is an integer.
- The sum of any two whole numbers is a whole number.
- All United States coins are silver-colored.

11. An athletic field is a rectangle, 120 yd by 60 yd, with a semicircle at each of the ends. A running track 15 yd wide surrounds the field. How many yards of fencing do you need to surround the outside edge of the track? Round your answer to the nearest tenth of a yard. Use 3.14 for \( \pi \).

12. What is the next number in the pattern?

\[ 1, -4, 9, -16, \ldots \]

13. The base of a rectangle is 7 cm less than three times its height. If the base is 5 cm, what is the area of the rectangle in square centimeters?

14. How many cubes would you need to build the structure shown below?

15. The measure of an angle is three more than twice its supplement. What is the measure of the angle?

16. A square and rectangle have equal area. The rectangle is 32 cm by 18 cm. What is the perimeter of the square in centimeters?

17. What is the value of \( x \)?

\[ \left( \frac{5}{3} + 5 \right)^\circ \quad (2x - 70)^\circ \]

18. What is the \( y \)-coordinate of the midpoint of a segment with endpoints \((0, -4)\) and \((-4, 7)\)?

**Short Response**

19. Write the converse, inverse, and contrapositive of the following statement. Determine the truth value of each.

If you live in Oregon, then you live in the United States.

20. \( \overline{AB} \) has endpoints \( A(3, 6) \) and \( B(9, -2) \) and midpoint \( M \). Justify each response.

- a. What are the coordinates of \( M \)?
- b. What is \( AB \)?

**Extended Response**

21. The sequence below lists the first eight powers of 7.

\[ 7, 7^2, 7^3, 7^4, 7^5, 7^6, 7^7, 7^8, \ldots \]

a. Make a table that lists the digit in the ones place for each of the first eight powers of 7. For example, \( 7^4 = 2401 \). The 1 is in the ones place.

b. What number is in the ones place of \( 7^{34} \)? Explain your reasoning.

22. Write a proof.

**Given:** \( \angle 1 \) and \( \angle 2 \) are supplementary.

**Prove:** \( \angle 1 \) and \( \angle 2 \) are right angles.